Short hints on the summer assignment for Calculus BC, 2019-2020

1. Let $j(x) = 5e^{3x-2}$.

   a) Find the domain and range of $j(x)$.
   The domain is the set of values you can enter into a function; the range are the values that come out. For this course, the largest either can be is the set of real numbers.

   b) Let $k(x) = \ln(x) - 3$. Find $(j \circ k)(x)$. Simplify.
   This is a composite function.

   c) Find $j^{-1}(x)$.
   Either undo each action in the opposite order, or "reverse the roles of x and y," and solve for y.

2. **a)** Divide the polynomial $x^3 + 2x + 3$ by $x^2 + 2x + 3$.
   Divide just like regular long division. Be careful and line up your columns for each power of x.

   b) Find the asymptotes of the function $f(x) = \dfrac{x^3 + 2x + 3}{x^2 + 2x + 3}$.
   When is the denominator zero? What happens as x gets very large? Look at your previous answer.

3. The table below gives the values of a few functions at several points.

x	-2	-1	0	1	2
$w(x)$	32	25	18	11	4
$k(x)$	3	6	12	24	48
$r(x)$	0	-3	0	9	24
$s(x)$	17	24	17	10	17

   a) The function $w(x)$ is a linear function. Find an equation for $w(x)$.
   There are easier means, but you could write down a general equation for a line, plug in a couple points, and solve for the constants.

   b) The function $k(x)$ is an exponential function. Find an equation for $k(x)$.
   There are easier means, but you could write down a general equation for an exponential function, plug in a couple points, and solve for the constants.

   c) The function $r(x)$ is a quadratic function. Find an equation for $r(x)$.
   There are easier means, but you could write down a general equation for a quadratic function, plug in a three points, and solve for the constants.

   d) The function $s(x)$ is a sinusoidal function for which the maximum and minimum values are shown. Find an equation for $s(x)$.
   There are easier means, but you could write down a general equation for a sinusoidal function, plug in a few points, and solve for the constants. A general sinusoidal function is a sine curve that may have the x or y values "rescaled" and may have horizontal or vertical shifts. The equation might be written as $s(x) = A\sin[B(x - h)] + k$.

4. Solve for x: $\dfrac{2}{x} - \dfrac{2}{1+x} = 1\,.$
   Find a common denominator or clear the denominators.

5. Solve $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = \dfrac{8}{3}$ for $-\pi < x < \pi\,.$
   Again with the denominators. You might want to use a double angle formula.

6. Complete the square to find the vertex of the parabola $q(x) = 3x^2 - 6x + 7\,.$
   Proper form is to complete the square "on one side," by adding and subtracting the same value.

7. Evaluate $\displaystyle\sum_{n=2}^{5}\left(n^2 - \frac{1}{n!}\right).$
   The symbol $n!$ refers to n factorial, the product of the first n positive integers.

8. Evaluate the following limits. You are expected to show your work in the calculation of these limits. If a limit does not exist, then indicate that.

   (a) $\displaystyle\lim_{x\to 0} x^2\cot^2 3x$

   You will want to remember that $\displaystyle\lim_{x\to 0}\frac{\sin x}{x} = 1.$

   (b) $\displaystyle\lim_{x\to -3}\frac{x^2-3x-10}{x^2-2x-15}$

   Factor.

   (c) $\displaystyle\lim_{x\to 5}\frac{x^2-3x-10}{x^2-2x-15}$

   Again, factor.

   (d) $\displaystyle\lim_{x\to -\infty}\frac{x^2-3x-10}{x^2-2x-15}$

   Think about asymptotes. If you don't see the answer, then divide everything by x^2 and see what happens.

   (e) $\displaystyle\lim_{\theta\to 0}\frac{2\sin\theta - \sin 2\theta}{\theta^3}$

   Start with a double angle formula, and remember $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}.$

9. Solve for x:

$$\sqrt{8}\cdot 16^{3-x} = \frac{\sqrt[3]{2}}{4^{x-1}}.$$

   I start this problem by changing everything to the same base (2), and then using laws of exponents.

10. Let $f(x) = x - 2x^3$. Find $\displaystyle\lim_{x\to 5}\frac{f(x)-f(5)}{x-5}.$

    Ye olde difference quotient. Just be careful with your order of operations. Factor, if necessary.

11. Draw the graph of a function $j(x)$ that is defined for all real numbers and has the following features: $\lim_{x\to 3^-} j(x) = 2$, $\lim_{x\to 3^+} j(x) = 5$, and $\lim_{x\to 1} j(x)$ does not exist.
Note all of the restrictions, particularly that j is defined for all real numbers! The first two limits refer to the limits as x approaches 3 from the left or from the right.

12. Let $g(x) = \dfrac{1}{\sqrt{2-x}}$. Find $\lim_{h\to 0} \dfrac{g(x+h)-g(x)}{h}$.

    Another difference quotient. Evaluate $g(x+h)$ properly before simplifying.
    N.B.: $g(x+h) \neq g(x) + h$!!

13. Carbon-14 decay (radiocarbon dating) is used in archaeology to measure the age of ancient pottery. Carbon-14 has a half-life of 5830 years. How old is a pot in which 20% of the Carbon-14 has decayed?
Write an exponential equation, plug in the known values to solve for the constants, and then answer the question.

14. The depth of water is measured at a bridge varies sinusoidally. At 3:00 pm it is high tide and the water reaches 8 feet deep. At 9:15 pm it is low tide and the depth is 3 feet. What was the depth of the water at 7:00 pm?
Find the amplitude, period, vertical shift, and horizontal shift. Put this together to write a general sinusoidal function, as in problem 3. Then answer the question.

15. A curve is defined parametrically by $x(t) = \sqrt{4-t}$ and $y(t) = \sqrt{t}$ for $0 \le t \le 4$. What is an equation for this curve in Cartesian (rectangular) coordinates? (Graph it!)

    You already had the hint to graph the function. Solve this problem by eliminating the parameter (t). You can do this by solving for t in one equation and plugging it into the other equation.

16. Which of the following functions are odd, which are even, which are invertible?

    a) $y = \cos^2 x$.
    b) $y = e^{-x^2/2}$.
    c) $y = 2x^3 + 6$.
    e) $y = \ln(3x^2 - 1)$.
    f) $y = \tan^{-1} x$.
    g) $y = \sinh x = \dfrac{e^x - e^{-x}}{2}$.

    A function is odd iff $f(-x) = -f(x)$, for all x. The graph of an odd is symmetric with respect to the origin. Likewise, a function is even iff $f(-x) = f(x)$, for all x. The graph of an even is symmetric with respect to the y-axis. ("iff" means "if and only if.")

17. A circle through the origin can be represented by the Cartesian equation
$(x-a)^2 + (y-b)^2 = c^2$, where a, b, and c are constants, and $a^2 + b^2 = c^2$.

a) Write parametric equations for the circle.

Think about how the functions sine and cosine are defined in a unit circle. Then add vertical and horizontal shifts.

b) Write a polar equation for the circle. (Solve explicitly for r.)

Again, how are the functions sine and cosine are defined in a unit circle? Let $x = r\cos\theta$ and $y = r\sin\theta$, substitute, and solve.